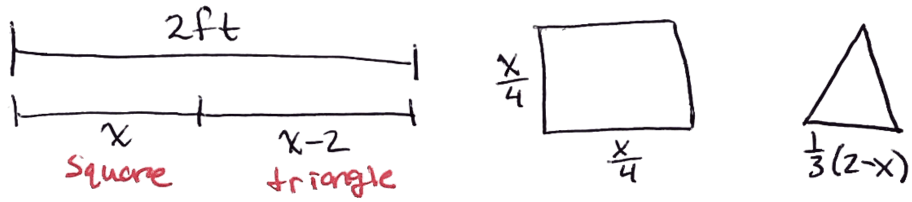
An Interesting Optimization Problem

Each time I teach Calculus 1 I usually get the question, "Why do we have to use the closed interval test, first derivative test, or second derivative test to check that our absolute max/min occurs at the critical number?" An example of why these tests are important is the question below. In this question, the absolute maximum of the total area function **does not** occur at the critical number of the function.

A two-foot piece of wire is cut into two pieces. One piece is bent into a square and the other is bent into a equilateral triangle. Where, if anywhere, should the wire be cut so that the total area enclosed by both the square and the triangle is maximized? The area of an equilateral triangle is $A = \frac{\sqrt{3}}{4}s^2$, where s is the length of one side. (You may give your answers as rounded decimals, if you prefer).



Total Area $= A = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(\frac{1}{3}(2-x)\right)^2 = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(2-x)^2, \; [0,2]$

$A' = \frac{x}{8} + \frac{\sqrt{3}}{36}(2)(2-x)(-1) = \frac{x}{8} - \frac{\sqrt{3}}{18}(2-x) = \frac{x}{8} - \frac{\sqrt{3}}{9} + \frac{\sqrt{3}x}{18}$

$\frac{x}{8} - \frac{\sqrt{3}}{9} + \frac{\sqrt{3}x}{18} = 0 \Rightarrow x\left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right) = \frac{\sqrt{3}}{9} \Rightarrow x = \frac{72\sqrt{3}}{9(9+4\sqrt{3})} \approx 0.8699$

Critical #

$x = \frac{8\sqrt{3}}{9+4\sqrt{3}} \approx 0.8699$

Closed interval test:

x	A
0	0.1925
$\frac{8\sqrt{3}}{9+4\sqrt{3}}$	0.1087
2	0.25 ← max

This means that the total area is maximized when $x=2$.

In other words, use all 2 feet of wire to make the square.